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MATHEMATICS IN THE TECHNOLOGICAL SCHOOLS OF COLLEGIATE GRADE IN THE UNITED STATES

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THE AMERICAN REPORT

COMMITTEE No. IX

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THE AMERICAN REPORT.

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I. INTRODUCTION.—PLACE OF APPLIED SCIENCE IN HIGHER EDUCATION IN THE UNITED STATES.

DEFINITIONS.

The most striking characteristic of technological as of other forms of education in the United States from the international standpoint is doubtless its extreme range of variation and the impossibility of framing general statements and definitions which shall be free from numerous exceptions. For the purposes of the present report a technological school or department may be defined as one giving systematic instruction in engineering and other forms of applied science to students entering the institution at the average age of 18 or 19, usually on the completion of a high-school course four years in length. This definition would exclude, for example, the lower-grade technical schools which admit younger students with less preparation, and which give the training necessary for minor industrial positions, with little or no use of the calculus; it would exclude, again, the evening schools for the supplementary training of men engaged in technical vocations. Neither of these important types is, however, as yet very much developed in the United States in comparison with the technological institutions of collegiate grade which are here discussed. It should be noted, however, that actual technological instruction is often preceded in the higher institutions, either from choice or requirement, by one or more years devoted to academic or general scientific studies; also that a considerable and increasing number of the technological students have previously completed a college course. The technological instruction aims to qualify its graduates both for immediate usefulness in junior positions in the various branches of engineering, in technical chemistry, in archi-

[1] There is some diversity of usage in regard to the use of the words "technology" and "technological." These terms, employed in the reports of the Bureau of Education and adopted by the commissioners in their general scheme, correspond with the titles of the various "schools" and "institutes" of technology, and are here used as nearly equivalent in meaning with "applied science" and "engineering," being, however, somewhat more definite and restricted in scope than the first and somewhat more comprehensive than the second. They seem to the writer to have for the present purpose the important advantage of comparative freedom from the vagueness or ambiguity which necessarily attaches to such words as engineering, technical, etc., which are in more general colloquial use. H. W. T.

tecture, and for ultimate professional responsibility proportionate to individual capacity, etc. The curriculum is usually four years in length (with a present tendency to extension), and students completing it commonly receive the degree bachelor of science (B. S.), with specification of the professional department. If the same institution gives also academic courses leading to the bachelor of arts degree, it is in general deemed, in what follows, to be a college or university having a technological department. The boundary can not, however, be very sharply drawn, and in some of the universities the technological department preponderates numerically.

HISTORICAL SKETCH.

Prior to the Civil War of 1861–1865 technological or engineering education in the United States was represented chiefly by the Rensselaer Polytechnic Institute at Troy, N. Y., founded in 1824; by the work of Union College in civil engineering; by the more recently established scientific schools of Harvard and Yale Universities; and in a special sense by the military and naval academies at West Point and Annapolis. July 2, 1862, a far-reaching act of Congress provided a national grant of public land to each State for the—

> endowment, support, and maintenance of at least one college where the leading object shall be, without excluding other scientific and classical studies, and including military tactics, to teach such branches of learning as are related to agriculture and the mechanic arts.

Some of these grants proved enormously productive; and subsequent acts have largely increased the resources of the institutions.

The rapid subsequent growth of technological education has been due not only to this legislation, but to the economic conditions of the times. The close of the Civil War was succeeded by a period of extraordinary industrial development, particularly in the building of railroads, but also in manufacturing and mining. The natural resources of the country were so abundant, the demand for immediate results so keen, the supply of trained engineers so scanty, that quick and wasteful methods were inevitable. The existing institutions and the methods of apprenticeship could no longer furnish a sufficient number of engineering recruits. Engineering had to be taught to large bodies of students by men apt to be inexperienced either in engineering or in teaching, sometimes even in both. The constantly expanding subject-matter had to be worked into teachable form by teaching it. Professional employment was so easily obtained that many students were unable to resist the temptation to curtail their preparation for it. Yet, on the other hand, there was widespread distrust of engineering education in its newer forms. Manufacturers accustomed to get on by rule-of-thumb methods were slow to realize the need of employing mechanical engineers and chemists. Educators trained in the classical tradition denied the educational value of utilitarian studies. Thus technological education during the past half century has had to work out its difficult problems sometimes in independent institutions, sometimes in connection with colleges and universities with, for a time at least, a conservative lack of sympathy for applied science. This relationship has greatly influenced the development both of technological education and of the colleges and universities which have undertaken it. At the present time, while its unsolved problems are numerous and pressing, its honorable status is established, and a great host of graduates are eagerly absorbed each year by the industries of the nation.

The fundamental subjects for technological students have naturally been mathematics, physics, and chemistry. Of these physics and chemistry had not been much developed in the older colleges, while a moderate range of mathematics had been widely employed as a means of mental discipline without much emphasis on its applications. The early students of engineering were naturally taught mathematics like their fellow collegians and by teachers of the same type. It was necessary for them, however, to attain some facility of mathematical technique; their time was limited, their appreciation of theoretical discussion not keen. The teachers of engineering, with vague recollections of their own collegiate mathematics (too often studied without reference to its applications), were apt to be successful in avoiding the use of anything beyond trigonometry, and thus confirmed the students in a willing belief in the relative uselessness of analytic geometry and the calculus to practical men. Out of these conditions have grown severe and sometimes exaggerated criticisms of mathematics and mathematicians, coupled with a tendency to insist that mathematics shall be taught merely as a "tool" and that the teaching shall be done by men of engineering training and experience.[1] There are indications of progress toward a better mutual understanding among the parties to this controversy. Recent active discussion will assuredly result in a better understanding and in progressive improvement.

The extraordinary development of applied science, the continual specialization of the engineering professions, the immense advances in the fundamental sciences, physics and chemistry, the necessity for turning out better and more broadly trained men, have in recent years caused severe congestion of the technological curriculum. This has been alleviated in some measure by condensation of subject matter, by time-saving methods of teaching, by utilization of the summer vacation for educational purposes, by concentration of attention on general principles at the expense of technical processes.

[1] Compare Symposium of Mathematics for Engineering Students, reprinted from Science, of July–September, 1908.

Nevertheless, it has often been found necessary to restrict the time assigned even to mathematics to very narrow limits, and there is a marked tendency among the stronger institutions to make the normal length of the course five years, or even six. The efforts to economize time have led, with other causes, to a critical revision of the traditional mathematical program, of which a fuller statement will be found below. These changes, while supported by a considerable body of expert opinion, are naturally still in the experimental stage. The fact that each higher institution in the United States has the full control of its own curriculum, with no attempt at standardization on the part of Nation or State, is naturally most favorable to the working out of such experiments.[1] Excesses in any direction are, on the other hand, likely to be checked by the inter-relations of the institutions and by the usually conservative influence of the larger and older among them.

PRESENT GENERAL CONDITIONS.

The Report of the United States Commissioner of Education for 1908 shows that of 101,000 collegiate students reported nearly 31,000 were taking engineering courses, 6,000 agriculture, and 12,000 general science. Of 120 universities, colleges, and technological schools included in the report as giving engineering instruction about one-third are wholly technological (or agricultural), and these have about three-eighths of the total number of engineering students.

The United States fosters technological education by considerable annual grants, amounting in 1907–8 to about $2,500,000, not including grants for the Military and Naval Academies. The control and management of the institutions are vested, however, in State (not national) officers, or in corporations which are sometimes wholly private, and sometimes include representatives of the State.

The relation of the several States to higher education varies widely. In the older eastern States instruction of collegiate grade is mainly carried on by great private institutions, some of which have received vast endowments from private sources. In the rest of the country, and particularly in the Middle West, the great State universities are liberally supported from the State treasuries. The national grants have in many cases been assigned by the States to the State universities; in some, however, to separate institutions.

The fundamental difference between the ordinary college (or the academic department) and the technological school or department consists in the definite professional aim of the latter. Here again there is a wide range from the technological school which confines

[1] In recent years certain private agencies, notably the Carnegie Foundation for the Advancement of Teaching, have come to exert a considerable influence in favor of relatively high standards by establishing conditions of eligibility for institutions deserving financial support.

its attention to technical subjects and those on which they directly depend, to the university which insists on two or more years of general college work as a preliminary to the professional. The independent agricultural and mechanical colleges have often maintained somewhat lower standards of admission and graduation.

While most universities and colleges leave their students at present a considerable freedom of choice of subjects, the engineering curriculum is generally laid out with relatively complete definiteness. During the first year or two the curriculum may be identical for all; thereafter the student follows the program he has selected, with perhaps some latitude in respect to the choice among collateral subjects or professional specialties.

The influence of university connection on technological education has various interesting aspects. When academic and technological departments are not very unequal in size and age, and when other conditions are favorable, each may react advantageously on the other. The development of work in applied science has greatly stimulated interest in the sciences themselves and in laboratory methods of instruction. University connection has in many cases insured a larger and stronger staff, a better library, a more favorable position for science departments, and a broader educational spirit and policy. In the larger technological schools many of these advantages have also been realized, together with somewhat greater freedom of development.

On the other hand there is danger that in a university the balance may not always be evenly held between the interests of the academic and the technological students, or there may be an excess of emphasis on abstract science, or the professional spirit of the technological student may suffer impairment from the distractions of academic athletics, etc.

An interesting review of present conditions, and particularly of current criticism of the teaching of mathematics to students of engineering, may be found in the reports of the joint meeting of the sections on mathematics and on mechanical science and engineering of the American Association for the Advancement of Science and the Chicago section of the American Mathematical Society, reprinted from Science of July 17, 24, 31, August 7, 28, and September 4, 1908, and referred to in the following pages as the Chicago symposium. Reference will also be made to a symposium on instruction in calculus at the Madison (1910) meeting of the Society for the Promotion of Engineering Education, of which, however, only a preliminary publication is yet available.

A committee appointed at the Chicago meeting to report to the Society for the Promotion of Engineering Education has courteously placed material collected by it at the disposal of the present committee. The scope of its work is indicated by the following preface of its report (which is to be published in the bulletin of the society):

The object of this report is to present a synopsis of those fundamental principles and methods of mathematics which, in the opinion of the committee, should constitute the minimum mathematical equipment of the student of engineering.

It is hoped that this report may be serviceable in two ways: First, to the teacher, as an indication of where the emphasis should be laid; and secondly, to the student, as a syllabus of facts and methods which are to be his working tools. It does not include data for which the student would properly refer to an engineer's handbook; it includes rather just those things for which he ought never to be obliged to refer to any book, the things which he should have constantly at his fingers' ends.

The teacher of mathematics should see to it that at least these facts are perfectly familiar to all his students, so that the professor of engineering may presuppose, with confidence, at least this much mathematical knowledge on the part of his students. On the other hand, if the professor of engineering needs to use, at any point, more advanced mathematical methods than those here mentioned, he should be careful to explain them to his class.

The defects in the mathematical training of the student of engineering appear to be largely in knowledge and grasp of fundamental principles, and the constant effort of the teacher should be to ground the student thoroughly in these fundamentals, which are too often lost sight of in a mass of details.

The committee has not found it possible to propose a detailed course of study. The order in which these topics should be taken up must be left largely to the discretion of the individual teacher. The committee is firmly of the opinion, however, that whatever order is adopted, the principal part of the course should be problems worked by the students, and that all these problems should be solved on the basis of a small number of fundamental principles and methods, such as are here suggested.

What is most needed at the present time is a series of synoptical textbooks, which shall present, in compact form, first, the fundamental principles of the science, and second, a classified and graded collection of problems (which would naturally be subject to continual change and expansion). It is the hope of the committee that this report, which is confined to the first part of the desired textbook, will stimulate throughout the country practical contributions toward the second.

II. PLAN OF INVESTIGATION.

A detailed questionnaire (reprinted below), prepared by the present committee in cooperation with that on colleges of liberal arts and universities, State and endowed, was issued by the United States Commissioner of Education in December, 1909, to a large number of institutions believed to be representative. The accompanying reports of subcommittees, one on technological schools, the other on technological departments, are based largely on a comparative study of the replies received. No attempt has been made, however, to secure statistical completeness. The remainder of the present report consists mainly of a review of the subcommittee reports with some comparative discussion of the facts and conditions brought out in them.

The committee tenders its grateful thanks to the Commissioner of Education for his valued aid, and to the fellow mathematicians to whose painstaking and suggestive replies these reports owe much of whatever value they may possess. Special mention should also be made of the cooperation of Professors A. Ziwet, of the University of Michigan and E. R. Hedrick, of the University of Missouri.

Questionnaire on the Teaching of Mathematics in Universities, Colleges, and Technological Schools or Departments.

General Information.

1. Name of institution. Place.

2. Mathematical staff (complete list with present titles, academic degrees, and a brief mention of any special preparation of members, e. g., training in physics, astronomy, or technology, or engineering practice).

ORGANIZATION.

3. List of departments (schools, colleges, etc.) in which mathematics of collegiate grade is taught.

4. List of degrees given in these departments for undergraduate work.

5. In these different schools or departments are there separate mathematical departments? If not, what combinations are made? What relations subsist between different mathematical departments? (Compare with question 2.)

6. Status of mathematical department in its relations—dependence or independence—to the general faculty or to other coordinate departments.

7. Status of mathematical teachers as to internal organization of mathematical department; influence or control exercised by senior teachers or head of department over junior colleagues.

8. To what extent are members of the department restricted or directed by administrative officers in the conduct of their instruction, either as to subject matter or methods?

9. What principles govern distribution of teaching? For example: Do senior teachers give a large or small proportion of elementary or required instruction, and are advanced elective subjects intrusted to junior teachers?

10. Are department meetings held regularly for discussion of current teaching, or scientific matters, or both?

11. To what extent are collegiate and technological students taught in the same classes, and what are the reasons for or against such combinations? (Compare 26.)

12. To what extent are mechanics and descriptive geometry or other branches of applied mathematics taught by the mathematical department?

13. By what system or method are vacancies in the mathematical staff filled?

14. What material changes, if any, in matters referred to in questions 1 to 13 have been made during the past 10 years?

15. What tendencies to change are now in operation, and what changes seem to you desirable?

CURRICULUM.

16. Extent of admission requirement. (Preferably in terms of the definitions recommended by Committee of American Mathematical Society and now in use by the College Entrance Examination Board. In case of deviation from these, a brief statement of reasons is desired.) Samples of examination papers or certificate blanks are requested. Age of students at entrance?

17. General quality of mathematical preparation as indicated by the results of your admission system? A brief discussion of prevalent defects in preparation is invited.

18. What changes of admission requirements in mathematics have been made during the past 10 years, and what changes are still desirable?

19. Outline of mathematics curriculum, with brief statement of conditions which have determined it, and of any existing tendencies to change it. It is desired that time and number of exercises allotted to each subject be shown; that textbooks in use be mentioned, and any deviation from usual selection or order of topics be stated, with reasons. The extent to which subjects ordinarily separate are combined is of particular interest. Status of mathematics, as required or elective subject, as a major or minor subject, as requirement for degrees, or for recommendation for secondary teaching.

20. What mathematical courses are required or elective in advance of the (first) course in calculus? Does an extension of this list either as to required or elective subjects seem to you desirable and probable for technological students? Do you favor, for example, mathematical requirements equal to those of the continental technological institutions?

21. To what extent is the curriculum determined either as to content or aim by the needs of other departments dependent on mathematical preparation?

22. To what extent is there a tendency in emphasizing applications of the mathematics to introduce elementary mechanics, physics, thermodynamics, etc., in the courses in mathematics?

23. How far and in what manner are mathematics and engineering, or allied subjects, correlated?

24. What is the attitude of professional and applied science departments toward pure mathematics as to time allotment, aims of instruction, etc.? Do those departments prefer to teach such mathematics as they desire?

25. To what extent is the mathematical curriculum restricted by congestion of subject matter in the engineering courses?

26. To what extent does the mathematical curriculum for technological students correspond with that for collegiate students? (Compare 11.)

27. What instruction is given in history of mathematics?

28. What changes have taken place in the matters covered by questions 19 to 27 during the past 10 years, and what changes are present tendencies likely to cause? In particular, any present or projected extension of the usual four-year technological course affecting the mathematical instruction should be stated.

Aims and Methods of Teaching—Examinations.

Brief description of aims and methods of teaching, indicating relative emphasis, and in particular—

29. Aims in teaching as to relative emphasis on different results—e. g., mathematical dexterity, analytical power, logical keenness, accuracy in computation. To what extent are our students—collegiate or technological—gaining from their mathematics either general culture or mathematical power and resourcefulness?

30. Extent to which lectures, oral recitations, blackboard work, written tests, criticism of student's work in class or privately, are used. Are so-called seminar methods found useful in teaching undergraduates? If so, in what subjects?

31. Are definite programs of elementary courses prepared in advance; and, if so, are they duplicated for teachers and students?

32. Manner in which records are made up, and extent to which they depend on term work or a final examination.

33. Degree of importance attached to independence in ordinary problem work, and steps taken to secure or test it. Is problem work criticized and returned? If so, by whom?

34. Extent to which personal conference of students with teachers is employed.

35. Normal amount of teaching for teachers of different grades, and extent of their incidental duties.

36. Average size of classes in principal subjects. If instruction is coeducational, state proportion of women students.

37. Use made of models. Are models made by students?

38. Is mathematical drawing or other work of laboratory character required or offered by the mathematical department?

39. To what extent are students allowed, or advised, to use tables of formulas in examinations or in the classroom?

40. What changes in methods of teaching have been made during the past 10 years, and what are present tendencies?

Preparation of Teachers.

41. What kind and extent of preparation seems to you most desirable for members of your staff? For example: An academic or a technological undergraduate course (or both) followed by graduate work in mathematics? Do you consider it practicable and desirable to secure teachers who either before or after their graduate work in mathematics have had actual engineering experience? What importance do you attach to previous teaching experience in college teaching? In secondary teaching? (See also question 13.)

42. Do you favor a systematic attempt on the part of technological schools or departments to prepare students for mathematical teaching? Should technological students of exceptional mathematical ability be induced to specialize mainly in mathematics?

43. Is it your opinion that colleges or technological schools should provide for undergraduate courses in the pedagogy of mathematics?

44. State, if possible, the total number of different persons who have given instruction in your mathematical department, or departments, during the decade 1899–1909.

45. Do you favor permanent or protracted retention of teachers in rank below that of assistant professor?

III. REVIEW OF REPORTS OF SUBCOMMITTEES ON INDEPENDENT TECHNOLOGICAL SCHOOLS AND ON TECHNOLOGICAL DEPARTMENTS OF COLLEGES AND UNIVERSITIES.

This review is based in part on the actual subcommittee reports, in part on other sources. While the subcommittee reports are intended to be primarily descriptive accounts of existing conditions and tendencies, with incidental expressions of opinion, the present review undertakes to deal with the subject in a somewhat broader general manner in the light of whatever information and experience are at the disposal of the committee, but always with reference to current conditions.

ORGANIZATION AND STATUS OF MATHEMATICAL STAFF.

The size, personnel, and proportion of men in different grades are in each institution a matter of more or less complex evolution, affected by many personal considerations. The mere size of the mathematical staff depends for its significance mainly on the relation it bears to the number of students, a relation not easily determined in the case of the institutions having a single mathematical staff for academic and technological students. In the judgment of the committee a department should normally have not less than half its members of professorial grade, including assistant professors. Comment on the qualifications desirable in junior teachers will be made later.

The departmental organization and the relation of the mathematical to the other departments appear to be relatively simple and elastic, as is naturally desirable. The extent of departmental organization existing or needful naturally varies widely with the size of the department and with the range of experience and attainments among its members. It appears to be generally recognized, as it should be, that men worthy of appointment even as junior teachers are entitled to a high degree of freedom as to methods of teaching. On the other hand when large classes are distributed in small sections among a number of teachers there should be control or approximate agreement as to subject-matter, aims, and rate of progress.

The very limited extent to which theoretical mechanics and descriptive geometry are taught in the mathematical departments seems to the committee unfortunate, and will be discussed in connection with the mathematical curriculum below.

ENTRANCE REQUIREMENTS IN MATHEMATICS.

The usual entrance requirements in mathematics for technological schools and departments include elementary algebra and plane and solid geometry. A considerable number of institutions require also plane trigonometry, a few add advanced algebra, and a few arithmetic. The following definitions of the more usual subjects, prepared by a committee of the American Mathematical Society in 1903, have been adopted by numerous institutions and by the College Entrance Examination Board.

Elementary Algebra.

The four fundamental operations for rational algebraic expressions; factoring, determination of highest common factor and lowest common multiple by factoring; fractions, including complex fractions, ratio and proportion; linear equations, both numerical and literal, containing one or more unknown quantities; problems depending on linear equations; radicals, including the extraction of the square root of polynomials and of numbers; exponents, including the fractional and negative; quadratic equations, both numerical and literal; simple cases of equations with one or more unknown quantities, that can be solved by the methods of linear or quadratic equations; problems depending upon quadratic equations; the binomial theorem for positive integral exponents; the formulas for the nth term and the sum of the terms of arithmetic and geometric progressions, with applications.

It is assumed that pupils will be required throughout the course to solve numerous problems which involve putting questions into equations. Some of these problems should be chosen from mensuration, from physics, and from commercial life. The use of graphical methods and illustrations, particularly in connection with the solution of equations, is also expected.

PLANE GEOMETRY.

The usual theorems and constructions of good textbooks, including the general properties of plane rectilinear figures; the circle and the measurement of angles; similar polygons; areas; regular polygons and the measurement of the circle. The solution of numerous original exercises, including loci problems. Applications to the mensuration of lines and plane surfaces.

SOLID GEOMETRY.

The usual theorems and constructions of good textbooks, including the relations of planes and lines in space; the properties and measurement of prisms, pyramids, cylinders, and cones; the sphere and the spherical triangle. The solution of numerous original exercises, including loci problems. Applications to the mensuration of surfaces and solids.

These definitions, eliminating minor variations in the existing definitions of different colleges, were framed with a view to simplifying the elementary algebra by omitting certain non-essentials, and at the same time enriching it by the introduction of graphical methods and of more numerous problems. At present an attempt is in progress to define geometry less vaguely by means of a standard syllabus.

The current unit-scale of the Carnegie Foundation and the College Entrance Examination Board rate these subjects as follows:

Algebra 1½, plane geometry 1, solid geometry ½, trigonometry ½, a unit being supposed to represent a year's work in the school with four or five exercises per week. The total admission requirements of a college or technological school of normally high standards amount to 14 or more such units.

The few institutions requiring advanced algebra differ very widely in their definitions of it, and it appears to be the general opinion that this subject is within the proper range only of the exceptionally able student in an exceptionally strong preparatory school. As to plane trigonometry as an entrance requirement there is some difference of opinion. It is generally recognized that the concrete trigonometry with its solution of practical problems is quite within the grasp of preparatory students and of decided interest to them in its application of the combined resources of geometry, arithmetic, and algebra. The same can hardly be said, however, of those theoretical portions of the subject so important for later mathematical work. It is also important to bear in mind that in the average secondary school sufficient emphasis will rarely be laid on accuracy in computation for the needs of the future student of engineering. Some institutions prefer that all the time available for mathematics in the high school should be devoted to the maximum degree of thoroughness in the elementary algebra and geometry rather than to a program including trigonometry also. Wherever elementary trigonometry can be completed without sacrifice of such thoroughness, it is probably desirable that it should be included. It does not appear that any increase in mathematical entrance requirements beyond elementary algebra, plane and solid geometry, and plane trigonometry, as above defined, is either probable or desirable.

As to a specific requirement in arithmetic, the vital importance of accurate computation and the marked inability of entering students in this direction are generally recognized. Only a few institutions, however, seek a direct remedy for this inability by an entrance requirement in arithmetic; some expect the examination in trigonometry to be a more or less adequate test; some include numerical computation in the algebra and geometry papers. The efficacy of any of these methods is limited.

As to defective quality of preparation, the causes are probably far too deep-seated to admit of anything more than gradual amelioration. The demand for preparation in mathematics is so large that the elementary teaching must be often entrusted to persons not specially qualified for successful mathematical teaching either by natural endowment, interest, or training. The financial rewards of the teachers are usually small. On the other hand there has been a marked improvement in textbooks, in the establishment of periodicals for secondary teachers, and particularly in the organization of local associations of teachers of mathematics—including both school and college teachers—for the discussion of professional topics.

As to the school programs, it is obviously important that the study of mathematics should not have been discontinued during the final year of preparation. A candidate who was adequately prepared a year or more before entrance may have lost much during the interval, and such losses are probably more serious in mathematics than in most other entrance subjects on which the dependence of later work is less direct.

MATHEMATICAL CURRICULUM.

In all the institutions mathematical instruction is continued from the lower limit determined by the respective entrance requirements, with more or less review overlapping, through at least a first course in the calculus, thus including additional algebra, trigonometry (if not passed at entrance), plane analytic geometry, as well as the elements of the calculus. In the stronger institutions the elements of solid analytic geometry are usually added, and the above program is completed by the end of the second year, the calculus extending through the whole of this year; in them an elementary course in differential equations is also generally required, at least of students in mechanical or electrical engineering. Among subjects less frequently required spherical trigonometry and least squares may be mentioned. A considerable number of institutions, mostly universities, offer a wide range of advanced electives including, for example, projective geometry, Fourier's series, vector analysis, theory of elasticity, hydrodynamics, theory of functions, etc. Few undergraduate engineering students find time, however, in a four-year course for such electives.

An interesting analysis of the mathematical curriculum in 17 representative institutions may be found in Prof. E. J. Townsend's valuable contribution to the Chicago symposium. It is there shown that "the average number of recitations given to each subject for the 17 institutions is approximately as follows: College algebra 50, plane trigonometry 46, analytic geometry 80, and calculus 130," this last usually including also a short course in differential equations. The range of variation is notably wide, in algebra from 15 to 90 exercises, in plane trigonometry from 29 to 70, in analytic geometry from 55 to 110, in calculus from 70 to 180. In interpreting these figures it must, however, be borne in mind that the institutions differ not only in their entrance requirements but in the strictness with which these are enforced, that the boundaries of the various subjects are not invariable, and that a relatively small time allotment may be compensated by a large requirement of outside preparation, or by more efficient methods of teaching, as subdivision of classes into smaller sections, or otherwise. The length of recitations is also not uniform and it may well be doubted whether 6 recitations per week for 15 weeks can be considered exactly equivalent to 3 per week for 30 weeks, or 2 per week for 45 weeks. Only one of the 17 institutions specifies a separate course in scientific computation, and this as an elective.

There is no apparent tendency to extend the program of required mathematics for engineering students beyond elementary differential equations. The extension of the engineering course to five years would probably favor such changes as the introduction of more thorough and extended courses in calculus, in mechanics, etc., and on the other hand the encouragement of elective mathematics.

The total time assigned to mathematics naturally varies with the general condition of the curriculum as to congestion or the reverse, and with the attitude of the administration and faculty as to where the boundary shall be drawn between mathematics and applied science. The congestion referred to, together with the development of mathematical criticism, and other causes have tended to compel more or less revision of textbooks in which the following tendencies may be noted:

In college algebra, the omission or postponement of such topics as series, partial fractions, undetermined coefficients, and the close articulation of the subject with analytic geometry and calculus, by insistence on the use of graphs, and by the early introduction of the ideas of function and derivative;

In analytic geometry, a similar tendency to employ calculus methods and to broaden the range of curves studied;

In calculus, a much earlier introduction of the fundamental ideas of function and derivative, as already noted, diminished emphasis on such matters as indeterminate forms, the avoidance of defective proofs, and the use of a much wider range of applications, including such matters as velocity, acceleration, attraction, moment of inertia, etc., a process naturally continued in differential equations.

Two current tendencies already noted have particular interest:

1. A better correlation between mathematics and engineering branches by the introduction of numerous and varied applied problems, or in some cases by insistence upon a thorough course in mathematical mechanics, and

2. A unification of the mathematical curriculum involving the redistribution of the whole subject matter according to its difficulty or its needfulness as an auxiliary, with little or no reference to the traditional division of subject matter into algebra, analytic geometry, and calculus.

The former change in its limited form has become rather general. The latter is still in the experimental stage.

CORRELATION OF MATHEMATICS WITH TECHNOLOGICAL SUBJECTS.

The introduction of applied problems connects itself with a gradual change of attitude on the part of the mathematical teachers, with the increasing development of applied mathematics both inside and outside the technological institutions, and with the criticism by physicists and engineers of the older, more abstract mathematical teaching, as illustrated by the following quotations from a distinguished engineer in the Chicago symposium:

> Abstract mathematical studies, if pursued as a kind of intellectual calisthenics, may produce a pure mathematician, but they may unfit a man for practical engineering.
>
> Mathematics is used in engineering to express the quantitative relations of natural phenomena. The mathematician delights in the relations; he divorces them from the phenomena and gives them abstract expression, while the engineer is concerned with the natural phenomena; he demands the physical conception; the medium of expressing these relations is of secondary consequence.

Such passages might be multiplied indefinitely from recent discussion, oftentimes coupled unfortunately with remarkable exaggerations and not less remarkable remedial suggestions.

For the 1910 meeting of the Society for the Promotion of Engineering Education the following questions were issued by the president in advance:

> 1. What proportion of the students in engineering schools, who have completed the required work in calculus, are able to apply that knowledge and solve simple practical problems?
>
> 2. What proportion of the graduates of engineering schools, who receive instructions in calculus and in its applications to mechanics, thermodynamics, and engineering, are capable of using calculus as a tool, readily, with difficulty, or not at all? etc., etc.

The answers to these questions by members of the society are obviously rough estimates—the questions themselves not admitting of exact answers—and vary far too widely to have definite quantitative significance. They leave no doubt, however, as to the existence of a serious need. Even if, as is claimed by some of the mathematicians, the difficulty is largely due to the inherent conditions of student human nature, and to the disposition of some of the engineering teachers to expect the impossible, it is only the more important that mathematicians should cooperate in maintaining the best relation between their subject and those which follow it.

Mathematical teachers and textbooks have both made progress in this direction. The tendency of the best modern texts, particularly in the calculus, is to emphasize the power to analyze and state problems mathematically and to interpret the solutions. This tendency has naturally had its enthusiastic votaries who have gone to the extreme of concreteness, including problems not merely of mechanics but of technical engineering. The serious objections to this are that under ordinary time limitations such matter can only be taught at the cost of needful thoroughness in mathematical technique, and that the successful dealing with these problems presupposes wider acquaintance with engineering data than the students concerned can generally have. Whatever may be appropriate for the exceptional student of previous technical experience, the average graduate needs a solid foundation in mathematics as such, and in mechanics, before attempting to deal with engineering problems. When he has laid that foundation it is better economy of time for him to be turned over to the engineering specialist, who should in turn have a sufficient mathematical training to utilize all the mathematical power his students bring to him. If not thus utilized it speedily atrophies.

Something has been accomplished already, and more will probably be accomplished in this direction by including a larger number and a wider range of applied problems in the mathematical courses. It seems to the present committee, however, of great importance that the general course in calculus should be followed by a thorough course in mechanics presented from the mathematical standpoint. Mechanics is commonly taught to some extent as a branch of general physics, to some extent also under the title applied mechanics or otherwise by one or more of the engineering departments. The vital importance and fundamental position of the subject are certainly such as to make it wise to present it from all three sides, the mathematical, the physical, and the engineering, at whatever expenditure of time this may require. On the mathematical side the need may be met either by having a substantial course in theoretical mechanics given by the mathematical department, or by the existence of an independent department of mechanics with sufficiently mathematical tendencies. The choice between these alternatives may naturally depend more or less on personal and local consideration. Without such emphasis on the mathematical side of mechanics the continuance of a gap between mathematics and engineering may be regarded as inevitable.

The argument for a more mathematical treatment of descriptive geometry is similar in kind, but the difficulty and the need appear to be less serious in degree. It is not of course the opinion of the committee that either mechanics or descriptive geometry should be taught wholly in the mathematical department, but rather that each should be strengthened on the mathematical side.

The so-called "Perry movement" in England and certain phases of current mathematical reform movements in Germany and France have aspects in common with this increasing attention to applications of mathematics, but this is eminently a matter in which local considerations must have great weight. American conditions are so different from English, German, and French as to preclude exact comparison; much more the adoption of foreign models or methods. A swinging pendulum passes from one extreme to the opposite, and the best current opinion in this country is not in sympathy with changes so radical as some which have been advocated in England. It is cogently urged that while mathematicians must teach their subject to the engineering students always with due appreciation of the uses to be made of it, with due subordination to abstract theory, with due care to facilitate the transition from mathematics through mechanics to engineering, they should on the other hand never be led away from this fundamental and difficult task in an ineffective effort to teach a little engineering prematurely by employing illustrative problems which are not yet wholly intelligible on the technical side. Thirty years ago current mathematical texts in the calculus contained none but geometrical applications. Now such physical subjects as velocity, work, and attraction are not only introduced as applications, but they are introduced at a very early stage for the still more important purpose of making the fundamental conceptions of derivative and integral more completely intelligible and significant. This wider sweep of applications undoubtedly sacrifices something of the previous unity and formal simplicity of the subject. The gain in interest and significance to the student far more than offsets this.

An exceedingly important form of correlation, apparently overlooked or underestimated by some of the engineering critics of current mathematical teaching, is that it is at least as much their own duty as that of the mathematicians to teach the use of mathematics as a tool, or let us say, rather, as an instrument or method. The teacher of mathematics usually needs all the time he has to teach his own subject and to meet the engineering teacher halfway by teaching general nontechnical applications. It is the part of the teacher of engineering to familiarize himself with the mathematical subjects, the methods, and even the notation his students have learned. Then he can teach them how to use their mathematics with a success and completeness not possible to his mathematical colleague. It is as ineffective and uneconomical to require a mathematician to teach engineering as to require an engineer to teach mathematics. Each must aim at the best attainable correlation with the other.

UNIFICATION OF THE MATHEMATICAL CURRICULUM.

The recasting of the series of elementary subjects into a consecutive course in mathematics presents many interesting aspects. In a striking passage of his Elementarmathematik vom höheren Standpunkte aus. I, 1908, Prof. Felix Klein draws an interesting contrast between two systems of mathematical thought and development. In the first—

a particularistic conception of mathematical science is fundamental, dividing the whole into a series of well-defined provinces. In each of these one seeks to gain his ends with the minimum use of outside means or dependence on allied branches. The ideal is a structure of each division beautifully crystallized out and logically complete in itself.

The other system, on the contrary—

lays the chief emphasis on an organic interrelation of the separate branches, and on the numerous suggestions they afford each other. It prefers, accordingly, the methods which open up the simultaneous understanding of several branches from a unifying standpoint. Its ideal is the comprehension of all mathematical science as of one entity.

Recognizing the great part which each system has played in the development of mathematics he earnestly deprecates the predominance of the former in the mathematics of the schools, under the influence of Greek geometry and of the great eighteenth century analysts, who developed algebraic theories of infinite series and the like, with careful avoidance of the newer calculus methods.

The recent experiments in unifying the mathematical curriculum of the technological course represent a reaction against this prevalence of the first system distinguished by Klein. In discussing them it may be noted first that a similar tendency has some time since led to the introduction of the elements of analytic geometry in the form of graphs into the preparatory instruction in elementary algebra, and that there is a tendency in recent textbooks of elementary geometry to depart more widely from strict Euclidean traditions. The revision of the more advanced program has sometimes taken the partial form of merely eliminating college algebra or analytic geometry as a distinct subject. In one recent case the process has been carried further by combining the algebra, analytic geometry, a first course in calculus, and elementary differential equations into a single course and text-book.

Among the characteristics of this revision are:

The earlier introduction and consequently more prolonged study of the calculus ideas and methods, function, derivative, etc.;

The treatment of analytic geometry rather as a general mathematical method than as a separate subject, with less emphasis on the conic sections and more on the geometric interpretation of a great variety of equations, including parametric forms;

The consecutive discussion of functions of one variable by the methods of algebra, analytic geometry, and calculus;

The subsequent combination of solid analytic geometry and partial differentiation in a briefer discussion of functions of two variables.

The working out of such a program is necessarily attended with some sacrifice of formal simplicity, and with occasional apparent loss of continuity. The advocates of it believe that such a thorough revision has, by testing values and eliminating non-essentials, effected considerable economies in time, and that the advantages of the plan will decidedly outweigh these defects, particularly as more prolonged experience facilitates the actual teaching. The student should for example gain power of discriminating attack by the use for each problem of the method best adapted to its solution. Such a program may prove less easy both to teacher and student, but may at the same time be none the less worth while. The chief difficulties to be guarded against are apparent or excessive discontinuity of subject matter, and the corresponding dispersion of interest and attention on the part of the student. He finds it difficult to grasp the unity of mathematical analysis, and easy to lose the thread in passing from the methods of algebra to those of calculus and then of trigonometry. It must be borne in mind, however, that this sort of transition is just what must be easily made in any successful application of mathematics, and that the relative ease of completing a somewhat arbitrarily defined subject and then passing to the next may be too dearly purchased. The result of the experiments in this direction will certainly prove of much interest.

In comparison with the current reform movement in Germany, which also aims at increased and early emphasis on the ideas of function and derivative, the American institutions have a certain compensating advantage for the more limited mathematical preparation of their entering students, in the fact that the higher institution here is able to determine the form and content of mathematical work farther back than would be the case abroad, from about the point indeed at which such changes would naturally take effect.

AIMS AND METHODS OF TEACHING—EXAMINATIONS.

Methods naturally depend upon aims, and a discussion of aims is essentially a discussion of the relation of mathematics as a part to technological education as a whole. The first essential aim of technological education is professional efficiency, which must naturally depend in some measure on command of mathematics as a tool. A less superficial analysis must recognize the distinction between immediate usefulness and ultimate efficiency, between training for subordinate positions and education for posts of large professional responsibility and leadership. A broader view must reckon with the fact that the student is to become an educated man and a citizen, not merely an engineer, and that his success even as an engineer will be incomplete if his education has been too narrow.

It is such considerations as these, recognized as they are both by the stronger institutions and by the leaders of the engineering professions, that must be reckoned with in any attempt to determine or to state the aims of mathematical teachings. In the limited time ordinarily available some degree of skill in mathematical technique, in computation, in the solution of equations, in differentiation and integration, some degree of power in applying mathematics to concrete problems must be given to all who are to complete technological courses. To the more capable student at least the teaching and study of mathematics should not, and does not, confine itself to these objects. To them there will come some recognition of the vitality of the science and of the inevitableness of its fundamental notions, as expressions of any exact thinking about phenomena, some due appreciation of the combined elegance and power of its methods, some ability to think mathematically. Such results can of course be accomplished only by teachers who are really mathematicians, and who therefore, unlike certain critics, see in their science something more than an elaborate computing machine or slide-rule. It is on teaching inspired by such aims that technological education must depend in part for that breadth and culture value which it ought always to have. The historical point of view should naturally find due recognition in this connection.

The methods employed in mathematical instruction tend in general to hold the student responsible for continuous study of a textbook and for the solution of many problems, with relatively little dependence on final examinations. Lectures are employed but sparingly, larger classes are divided into numerous sections, the average size in the stronger institutions not exceeding 15 to 20. The amount of class-room work per teacher in these institutions is ordinarily 12 to 15 hours per week, but in others may reach 20. With four or five small sections the teacher is expected to acquire personal knowledge of the individual student and his needs, and special provision for private conference is not infrequent. So-called "mathematical laboratory" work, the drawing of careful plots, and the making of models are precluded in general by lack of time, but the student's work in the laboratories of physics, mechanics, and engineering serves a similar purpose.

As to the use of tables of mathematical formulas for integration and other purposes, usage varies widely. Mathematical teachers are generally averse to encouraging the relatively mechanical use of such aids. It seems fair to suppose, however, that before the completion of the mathematical course students may advantageously be instructed in the use of such mathematical tables as will be of most future service to them, without which, indeed, their mathematics may remain largely unused.

PREPARATION AND QUALIFICATIONS OF TEACHERS.

The ideal teacher of mathematics for technological students, besides the qualifications desirable in teachers generally, needs on the one hand to be a mathematician by natural endowment and by training, on the other hand to have intelligent appreciation of the mathematical needs of future engineers, and sympathetic interest in their point of view. This naturally presupposes previous occupation with and some measure of attainment in applied mathematics. Capacity for mathematical investigation, while naturally desirable, is not indispensable, but some form of scientific productivity may well be insisted upon. The particular method by which the individual teacher has acquired the qualifications stated is of minor importance in comparison with the personal considerations involved. Under present educational and industrial conditions the supply of competent teachers will doubtless long continue to fall short of the demand; only a small minority of technological students will engage in teaching, and the number of men of engineering experience possibly available for mathematical teaching may be safely characterized as "an infinitesimal of higher order." In the judgment of the present committee the future teacher of mathematics to technological students should have taken an undergraduate course, either technological or including a substantial amount of physical science with laboratory work. Such a course should be strong in pure and applied mathematics, and should be followed by graduate study, also including both pure and applied mathematics, and leading in general to the doctor's degree. This plan is believed to be as simple and economical as possible, but by no means the only one for reaching the normal professional standard. The graduate work will in general be taken at a university, and it is important that the candidate should be enabled under the rules of the university to avail himself of opportunities for advanced work in mechanics, mathematical physics, or engineering with full credit toward his degree.

The importance of previous experience varies widely with individuals, and only the stronger institutions will be able to insist upon it for minor appointments. Experience in secondary teaching, if not too prolonged, may have particular value from its frequently greater systematization, and still more in giving the future college teacher intimate knowledge of the conditions and methods obtaining in the schools from which his students are to come.

IV. CONCLUSION.

The history of mathematics exhibits many alternations of interest between pure and applied mathematics. The needs of astronomers or physicists have stimulated the development of a certain branch of mathematics, the development has been continued by mathematicians with more or less indifference as to the applications until the vein has perhaps been worked out. At present in the United States the pendulum seems to be swinging toward increasing interest in applied mathematics, at any rate up to differential equations, with probability of resulting advantage both to mathematics itself and to the allied sciences. Toward this change the technological institutions and the development of technology have contributed much by presenting problems requiring the development of new mathematical methods, by creating a need for the thorough mathematical teaching of great numbers of students, and by promoting through their reaction on the universities and on the public a widespread interest in the applied sciences.

Not many technological institutions or departments have yet encouraged research in applied mathematics, or undertaken to train future teachers of mathematics. Not many technological students have capacity for mathematical research or inclination for mathematical teaching. It is fair to expect that as the stronger schools gain in resources, and as the excessive demand for their graduates slackens with the increasing supply, they will build up graduate departments in which advanced work in mathematics will be an important element. At the same time the graduate schools of the universities with more liberal recognition of applied science will attract a larger proportion of technological graduates.

There is evidence that the technological schools and departments are reacting from over-emphasis on purely professional training and efficiency in favor of the fundamentals of general scientific education. The leading universities already require two years of academic instruction preceding the technological. While it seems improbable that the completion of an academic course will become a prerequisite for admission to the technological, a considerable and probably increasing number of students are likely to follow this plan.

On the whole the situation of mathematics and mathematical teaching in the technological institutions, if not altogether satisfactory, may be regarded as at least relatively so, in the sense that needs are recognized and conditions are favorable for steady progress. There is a just basis for high expectations as to the future development of the science which has owed so much of its past and present vitality to the insistent demands of astronomers, physicists, and engineers. In meeting these expectations measurably, in building a solid foundation for their engineering students, and at the same time in doing their part in that development of the science, it may well be the unceasing endeavor of mathematicians in the technological institutions to use their limited time economically and efficiently, studying the needs of the applied science departments sympathetically, enlisting all possible reciprocal interest, maintaining the dignity and integrity of mathematical science, not insisting too much on its rigorous refinements.

SUBCOMMITTEE 1. INDEPENDENT TECHNOLOGICAL SCHOOLS.

The following report consists essentially of an analytical study of replies received from 22 institutions, wholly or mainly technological in their character, to the questionnaire issued on behalf of the commission by the United States Bureau of Education. The text of this questionnaire and a discussion of the general subject may be found in the above report of the main committee. Certain statements of fact, applicable alike to the independent schools and to the university departments, have also been relegated to the main report.

I. ORGANIZATION.

Questions 2–15: Mathematical staff, degrees, experience, etc.; divisions of institution and relations to each other; relation of individual teachers to department and to administrative officers; distribution of work; method of appointment; tendencies to change.

Mathematical staff.—An examination of the mathematical faculties in the different institutions shows that a large proportion of all the professors, associate professors, and assistant professors hold one or more degrees above the baccalaureate degree. Among the instructors about half appear to hold such advanced degrees. Nearly half of the full professors and about a quarter of the assistant professors and instructors hold the degree of doctor of philosophy. Less than one-half of the full professors are graduates of classical colleges; something less than half seem to be graduates of technological schools or of the scientific departments of universities, while it is impossible to tell from the other degrees, Ph. D., etc., whether those who hold them were originally graduates of colleges or of technological schools. The degrees would indicate that the associate professors are for the most part graduates of technological schools or the scientific departments of colleges. Among the assistant professors there seem to be more college than technological graduates, although, as in the case of the full professors, the degree of doctor of philosophy does not indicate the course pursued. The majority of the instructors are from technological schools, although quite a large number are from colleges. There are also a few who have the degree of doctor of philosophy, but this does not indicate the kind of institution from which they first graduated.

Among the full professors a very large majority, if not all, have done advanced, or specialized, work; probably more than half of them in mathematics and more than half of them in engineering or related subjects. The others have specialized in physics and chemistry. A large proportion of the assistant professors also have had university training, and most of these have specialized either in mathematics or advanced engineering work. The graduate work of instructors is stated in so few instances that it is impossible to make any general statement or draw any conclusions.

It is evident from the above statement that the technological schools expect professors and assistant professors to have done graduate work, usually in the line of mathematics, but the work in engineering is esteemed so desirable that a man who has specialized in that rather than in mathematics may be appointed to a mathematical assistant or full professorship. It is also evident that the institutions expect the men who hold full or assistant professorships to have carried their advanced work to such a point that they have received an advanced degree.

The first degree granted by the technological schools is usually bachelor of science (B. S.), with or without added specification of department. Several, however, give the professional degree civil engineer (C. E.), etc.

The mathematical department is wholly independent of other departments, but generally under the control of the whole faculty so far as the amount of time to be devoted to mathematics is concerned and sometimes as to the individual subjects taught.

The department as a rule has a head who exercises general control over the other instructors, but these instructors appear to be practically independent in the conduct of their classroom work.

The members of the department are not restricted by the administrative officers either in the subject matter or in their methods, except in case of two institutions where the subject matter is laid out and general methods suggested in a conference between the head of the department and the president.

In the majority of institutions the advanced subjects are taught by the older professors, but in three institutions the professors and assistant professors meet all classes; in two institutions work is assigned without reference to rank, but according to fitness; and in one institution advanced elective subjects, like differential equations, least squares, etc., are taught in the engineering departments.

In the majority of technological schools meetings of members of the department are held irregularly for the discussion of topics connected with current teaching work and in a few of the institutions regular meetings for the same purpose are held. These meetings are generally for no other purpose, but in the case of a few institutions general mathematical subjects are discussed. The tendency to hold such meetings is naturally stronger in the larger institutions where 10 or more men have charge of different sections of a class. In two institutions there are daily meetings for the teachers of the different sections of the same class.

In the majority of the technological schools the mathematical department teaches mathematics only. In a very few cases mechanics is taught, and in two instances descriptive geometry.

Vacancies are usually filled by the board of trustees upon the recommendation of the president. This is virtually an appointment by the president. In nearly all cases the head of the department is consulted by the president, and usually the head of the department nominates to the president either directly or through the dean.

In most of the institutions considered there have been no recent changes of organization, and no changes are deemed desirable. A few institutions express the wish that the different branches of mathematics might be better correlated, and some desire a closer correlation between the mathematical department and the engineering departments.

II. MATHEMATICAL CURRICULUM.

Questions 16–28: Entrance requirements; outline of curriculum; elective subjects; mathematical needs and attitude of other departments; correlation with allied subjects; time restrictions; tendencies to change.

Entrance requirements.—Nearly all the institutions admit students on the basis of high-school certificates. Nearly all require elementary algebra, plane and solid geometry for admission, the definition of these subjects corresponding with those recommended by a committee of the American Mathematical Society in 1903. (See page 14.)

In a few cases other mathematical work, usually plane trigonometry, may be offered.

Where a definite minimum age limit is given it is usually 16 or 17 years, but so far as the answers indicate the actual age of entrance is from 18 to 19, as for other collegiate courses.

As a rule the technological schools report that students are not well prepared in algebra and geometry. The students are said to be inaccurate, to be unable to reason, to have forgotten elementary principles. Several institutions call attention to the fact that algebra being given in the first two years of the high-school work, much is forgotten before the end of the course. One institution states that it must receive graduates of the high schools because it is a State institution. Some feel obliged to review high-school algebra in order to prepare the student for college algebra.

Few changes of entrance requirements have been made during the past 10 years, except that several institutions which did not formerly require solid geometry now do so. Several institutions suggest that trigonometry might be added to the requirements, but some of them say that it is so poorly taught in the high schools that its addition is questionable.

Curriculum.—College algebra, trigonometry, analytic geometry, and calculus are usually required of all engineering students. In a few cases the subjects taught under algebra are given in connection with analytic geometry and calculus. There seems to be quite a tendency to take up analytic geometry and calculus together, and in some places to begin their study early in the course. A few institutions offer elective mathematical subjects. In some cases special subjects are required in some courses; thus, differential equations may be required of electrical or mechanical engineers.

The textbooks used are the ordinary modern mathematical textbooks. In a few cases mechanics is taught in the mathematical department. In two institutions the mathematical work is very heavy, in one of these 7 or 8 hours being required per week during the greater part of the freshman and sophomore years; and in the other, from 9 to 15 hours per week during the first two years. In some instances the elements of spherical trigonometry are given to all students, while in others this work is reserved for the civil engineers.

Few institutions require mathematical work beyond calculus, but quite a number offer electives. Where more advanced work is required it is usually differential equations or mechanics. Most of the institutions do not desire to increase their requirements in this respect.

Nearly all the institutions agree that the course in mathematics has been determined very largely by the needs of the engineering departments. While the statement is not definitely made, it appears in many cases that the course of study has been laid out by the mathematical professors so as to give students as much mathematical training and power as possible, at the same time having in mind that these men are to become engineers and will need to apply their mathematics to engineering work.

There is a tendency to emphasize the applications of mathematics by introducing problems in mechanics, physics, etc. The extent to which this can be done depends upon how much physics, mechanics, etc., the student knows. None of the institutions represented claim to teach physics or mechanics in the mathematical department; a few of the departments distinctly say that they do not give these applications, as it is their business to teach mathematics.

It is difficult to tell from the answers just how much correlation there is between the department of mathematics and the various engineering departments. The impression gained from reading the answers is that there is not very much correlation, certainly not so much as would seem desirable. Simple engineering problems are sometimes given in connection with mathematical work, but at the time the work is given students often do not understand the fundamental principles of the engineering subjects.

The professional departments seem not to interfere with the time allotment for mathematics, nor in general with the aim of instruction, although they sometimes desire to have more applications included in the mathematical courses. There are one or two exceptions to this, where it is alleged that the professional departments expect the mathematical department to teach the principles of engineering and to make applications. The engineering departments do not wish to teach mathematics.

About half of the institutions report that there is restriction of time on account of the congestion in the engineering courses; that the time for mathematics has been reduced to a minimum; and that the mathematical departments can not teach the subjects as they would wish to.

In most institutions no formal instruction in the history of mathematics is given, but many historical points are discussed informally in the classroom.

There is nothing to indicate that many changes have taken place during the past 10 years, or that many changes are contemplated. There is no indication that any extension of the four-year courses will affect mathematical instruction.

III. AIMS AND METHODS OF TEACHING—EXAMINATIONS.

Questions 29–40—Aims: Dexterity, power, keenness, accuracy, resourcefulness. Methods: Lectures, recitations, blackboard, tests, criticism, records, independence of work, conferences, amount of work for teachers, size of classes, uses of tables, tendencies to change.

Aims.—It seems probable that in defining these aims the professors of mathematics have stated the results which they would like to accomplish rather than those which they are actually obtaining. Some say that they do not strive for mathematical dexterity at all; others state that dexterity and accuracy receive the most attention.

Several give their aims in teaching as follows: Analytical power, logical keenness, accuracy, mathematical dexterity. Several state that considerable culture and resourcefulness are obtained. The question has not been answered fully enough for satisfactory conclusions.

Methods.—It is evident that few lectures are given to undergraduates in the technical schools. Large classes are divided into small sections receiving independent instruction, but following a common program and taking the same final examination. Oral recitations are held almost every day, with a large amount of blackboard work. Written tests are frequent in all of the institutions. Student work is criticised in class freely, and in almost all of the institutions students have the opportunity to confer with the instructors.

Programs of elementary courses are prepared in advance in some of the larger institutions, but not to any extent in the smaller ones. These programs are intended for the use of the instructors, but are changed to suit conditions.

Records; examinations.—Final grades are usually based on term work and final examination, although in a very few institutions final examinations are not given. In some institutions the grade is made up of results of written tests, daily recitations, and final examinations. The weight given to each of these parts varies from tests four-fifths, final examination one-fifth, to daily work one-half, final examination one-half. Several institutions do not allow the final examination to count for more than one-third of the final grade.

Problem work.—About half the institutions seem to lay great stress upon independent problem work, and have problems corrected by student assistants or by instructors and returned to students. The answers from the other half would indicate that problems are not systematically criticized and that no special effort is made to secure independent work.

Conferences.—In the majority of institutions the mathematical professors have regular office hours, and students are encouraged or required to have conferences with the instructors. This is especially true with respect to poor students. In only a few institutions is it stated that there are no such conferences.

Amount of teaching.—About half the institutions require from 12 to 16 hours per week of teaching, the others from 18 to 20 hours. It is evident that professors and assistants professors often have considerable committee and other additional work. Instructors do not seem to have any committee work, but they are expected to give considerable time to correction of papers, etc.

Size of classes.—In only three institutions is the average size of sections above 25, while in quite a number of them it averages from 15 to 20. The majority of them do not have women students, and where

Models; drawing.—In the majority of cases models are used to illustrate problems in solid analytics and calculus, but in a few cases it is distinctly stated that no models are used. In very few instances are the models made by the students.

Mathematical drawing and other laboratory work are not required or offered by mathematical departments with the exception of one institution.

Tables.—About half the institutions allow their students to use tables of integrals, while a smaller number allow tables of logarithms, and some others. A few do not allow tables at all.

Changes.—In the majority of institutions there have been no radical changes during the past 10 years.

IV. PREPARATION OF TEACHERS.

Questions 41–45: There seems to be considerable difference of opinion as to whether an academic or a technological course forms the better preparation for an instructor of mathematics in a technological school. A number of institutions suggest that both should be required. Practically all, however, agree that graduate work in mathematics is necessary. Nearly all believe that engineering experience is very desirable, but that it is almost impossible to obtain. There seems to be a great difference of opinion also as to whether previous teaching experience is desirable, some of the answers indicating that it is, and others stating very positively that it is not especially desirable for a young instructor who can obtain his teaching experience in the technological school. Nearly all agree that experience in a secondary school is not desirable.

The majority of the answers would indicate that a systematic attempt should be made on the part of technological schools to prepare students for mathematical teaching, although quite a large number have expressed a contrary opinion. Several of the answers call attention to the fact that the salaries offered mathematical teachers in technological schools are so low that few students care to consider such positions. About half of the answers favor inducing technological students of exceptional mathematical ability to specialize in mathematics; but some of the answers are decidedly against this. This may be interpreted to mean that the professors in the technological schools believe that students of exceptional mathematical ability have great opportunities as engineers and should not be induced to specialize in mathematics, thus giving up engineering careers.

It is the general opinion that technological schools should not provide for undergraduate courses in the pedagogy of mathematics.

Tenure.—In the majority of institutions the total number of different instructors during the past 10 years has been about twice as great as the number of instructors now in the department. This shows of course that the younger instructors change after a few years of service, presumably for the sake of increased incomes.

There is no unanimity of opinion in regard to the protracted retention of teachers without promotion. About half the answers indicate a belief that instructors should be retained permanently or for indefinite periods, while the other half indicate the contrary. The reasons are not given. There is probably a feeling that a man ought not to be held as an instructor after a certain period of years, because the salaries are usually very low, and he ought to find some work where he would be better paid. Some of those who do not favor keeping an instructor for a protracted period imply that they think if an instructor is worth keeping he should be given an assistant professorship.

The Agricultural and Mechanical Colleges.

These institutions were for the most part founded under the act of Congress of 1862, to which reference has previously been made. They include some of the separate technological schools on the one hand, and some of the State institutions on the other. It is convenient to group by themselves, however, those which concern themselves exclusively with agriculture and mechanic arts, and in which agriculture is a relatively important element. The classification is, however, not at all precise. The following notes based on replies from eight institutions may serve as a comparison of these institutions with the foregoing.

The majority of the heads of the mathematical departments have had scientific or technological training, and a majority of the associate professors seem to be graduates of scientific or engineering institutions; none of the latter are doctors of philosophy, and one has no degree. Of the instructors, a majority have had technological training; none hold the doctor's degree. In several instances members of the faculty give instruction in more than one department.

In one institution mathematics and engineering make one department.

In most of these institutions the requirements for admission are considerably lower than those of the technological schools, but there is a tendency upward. When an age limit is mentioned it is usually 16.

The mathematical curriculum usually includes some elementary algebra and solid geometry, followed by the subjects usually given in the other institutions. In most of them nothing is required or offered beyond the first course in the calculus. The agricultural and textile courses include less mathematics than the engineering.

The assignments of teaching average relatively high, amounting in some cases to 20 or 25 hours per week. The same is true of the size of sections, which is usually 20 or 30.

SUBCOMMITTEE 2. TECHNOLOGICAL DEPARTMENTS OF COLLEGES AND UNIVERSITIES.

INTRODUCTION.

The following report is based primarily on the returns to the questionnaire prepared by the committee and sent out on its behalf by the United States Commissioner of Education, as stated in detail in the general report of the committee. These returns have been supplemented somewhat from other sources where such material seemed essential to illustrate the practical aspect of the question. In order to give full weight to the returns, this material is introduced in the form of a supplement to the main body of the report.

ORGANIZATION OF DEPARTMENTS.

The replies to the questionnaire indicate a marked uniformity in the mathematical departments represented with respect to organization as well as method and aim of instruction. The cause of this uniformity seems to be that technological departments of colleges and universities are very largely a direct outgrowth of the academic departments by which they were preceded. This is evidenced by the fact that in none of the institutions represented is there a separate department of mathematics for technology, the mathematical instruction for all courses being centralized in a single department and given by the same corps of instructors. It is also noteworthy that the mathematical department is, in general, independent of other departments, and not subjected to definite restrictions as regards subject matter or methods other than a general limitation as to the time allowance for the work.

In the smaller colleges no distinction is made between students of technology and others so far as mathematical instruction is concerned. In the larger institutions the engineering students are usually separated from the academic, chiefly for administrative reasons and not because of any essential difference in the method or content of instruction. The size of the sections into which large classes are subdivided is usually limited to 25 members, this number seeming to be considered the maximum which can be taught to advantage at one time by the ordinary blackboard and recitation method.

In the smaller institutions it is not considered necessary to hold regular meetings of the department. In the larger universities, however, such meetings are held with more or less regularity for the purpose of encouraging an esprit de corps among the teaching staff, and enabling the professor in charge to keep in touch with the work of individual instructors and the department in general.

From an examination of the degrees held by the members of the various mathematical faculties represented, several interesting facts are apparent. It is found that an engineering degree is exceptional, the degrees most commonly held indicating that the majority of instructors have had an academic training. There is also manifest an increasing tendency to employ only those who have shown special ability by taking a higher degree, preferably the Ph. D., implying specialization in pure mathematics. This fact is especially noteworthy in its bearing on the relations between the departments of mathematics and technology, considered later.

ENTRANCE REQUIREMENTS.

The returns show that it is the general custom to admit students to all departments of the college or university on presentation of a satisfactory high-school certificate. The mathematical preparation thus certified invariably consists of arithmetic, elementary algebra, and plane geometry, usually with the addition of solid geometry and sometimes plane trigonometry, as fully explained in the general report of this committee. Throughout the Middle West it is customary for the university to publish regularly an accredited list of high schools whose students are thus admitted on certificate without examination. The principals of the schools on this list are kept informed as to the requirements of the university and the progress of their graduates after entering, by a high-school visitor specially appointed by the university for this purpose. From the university standpoint this system is very effective, as it enables the university to secure a much more thorough and uniform grade of preparation than would otherwise be possible. From the high-school standpoint the system was formerly criticized as unwarranted dictation on the part of the university, by reason of the fact that only a small proportion of high-school students are preparing for university work. This objection is now obviated in all the larger high schools by offering a variety of courses, only one of which is college preparatory.

In some cases, especially where the system of accredited schools is not in vogue, the universities report that the high-school preparation in mathematics is deficient for university purposes. This deficiency is usually credited to the fact that high-school mathematics is ordinarily given in the first two years of the course, and that in the two years which intervene before entering the university much of the work is forgotten or obscured. Under the system of accredited schools, however, it is customary to specially strengthen the college preparatory course in this respect by thoroughly reviewing during the last year all the mathematics previously learned.

In this connection it is of interest to contrast the American system of teaching elementary mathematics for a short period and then dropping it completely with the continental method as exemplified by the French lycée and the German and Austrian Gymnasium, Real-Gymnasium, and Ober-Realschule, where the study of elementary mathematics is carried on continuously for eight or nine years.

MATHEMATICAL CURRICULUM.

The extent of the mathematical curriculum is largely a question of breadth and thoroughness of training, the subject matter being nominally the same in all the institutions represented. The customary program of instruction includes algebra, trigonometry, and analytic geometry for four or five periods per week during the freshman year, and the elements of differential and integral calculus for the same number of periods during the sophomore year. During these two years much of the student's work is academic, whereas at the end of this period it becomes almost wholly technical. Mathematics is thus classed as an academic subject, at least so far as instruction is concerned, although mathematical principles are of course universally recognized as the basis of technology. As a matter of fact, only a small proportion of the mathematics actually taught is ordinarily used in engineering practice or in the succeeding courses in technology. As evidence of this fact the following statement may perhaps be as conclusive as any single fact that could be mentioned.

In returning a purely technical article that had been submitted to him for publication, the editor of one of the largest and most influential engineering journals in this country wrote as follows:

> While we find this an admirable treatise on the subject, we regret that it is too highly mathematical for our columns, due to the fact that it presupposes a thorough understanding of the principles of the calculus. It is our experience that the average machine designer has no, or at least only a vague, idea of the use of calculus.

This statement should perhaps be regarded as a criticism of the engineering profession in this country rather than as evidence that mathematical instruction is inefficient, or that a satisfactory technical education can be built up on such a limited mathematical basis as here implied.

This condition of affairs is no doubt as true for civil as for mechanical engineers. In electrical engineering, however, a somewhat higher degree of mathematical attainment is demanded, which is usually recognized by including differential equations as a required subject in this course.

The fact that ordinarily more mathematics is taught than actually used is sufficient reason why the returns to the questionnaire indicate no tendency to extend the mathematical requirements or to encourage electives in this subject.

CORRELATION OF DEPARTMENTS.

In the majority of institutions the needs of technological departments are at least nominally considered in arranging the work of the department of mathematics. Further than this there is no correlation of departments in any of the institutions represented in the returns. In a few of the smaller institutions the departments of mathematics and engineering are administered by the same head for economic reasons. In one instance it is specifically stated that not only is there no attempt at correlation, but that mathematics is taught as a separate discipline without reference to its technical applications.

UNIFICATION OF THE CURRICULUM.

Within the mathematical departments themselves there seems to be a general tendency toward rearranging the subject matter into a more compact and unified course. Effort is directed toward obliterating the formal boundaries which have been raised between the various subjects in the course of their historical development, and to construct a single course in mathematics in place of a succession of separate and somewhat unrelated courses in algebra, trigonometry, analytic geometry, and calculus. The most notable instance of this experiment is that being carried out at the Massachusetts Institute of Technology.

CORRELATION OF MATHEMATICS AND TECHNOLOGY.

The consensus of opinion on this topic is that in mathematical courses mathematics should be taught rather than applied science. It is generally believed that the introduction of technical applications in the mathematical classroom is advisable only when they are so elementary and familiar as not to destroy the integrity of the mathematical courses or diminish their educational value by obscuring general principles. All agree that no illustration should be used which requires a technical knowledge not possessed by the student or readily explainable to him, as the introduction of unfamiliar concepts merely offers the student two difficulties to surmount instead of one. The chief difficulty seems to be in finding technical illustrations which are sufficiently familiar, as the mathematical courses are confined to the first two years of the curriculum, and thus precede the student's professional training.

In one instance the objection is raised that by enlarging applications the concrete tends to fill so large a place in the mind that the principle in hand is obscured as one of general application. The general opinion, however, is that if it is possible to arrange courses in parallel, so as to furnish material for illustration, a training in formal mathematics in connection with its technical applications gives depth of understanding, strength of grasp, and freedom of use most conducive to the production of efficient and progressive students.

ATTITUDE OF DEPARTMENTS OF TECHNOLOGY.

The majority of replies state that technological departments are satisfied to leave the mathematical preparation of their students to the department of pure mathematics as at present constituted. In a few instances it may be inferred from the returns, however, that technological departments are not wholly satisfied with the present preparation in mathematics as a basis for their work and would prefer to have the subject taught by men more familiar with its technical applications. In connection with this, there is a tendency to diminish the amount of pure mathematics required of technological students, one reply going so far as to say that the subject (as now taught) is regarded by engineers as a necessary evil.

There is a substantial agreement that there is no reason why engineers should be given essentially different elementary training in mathematics from others, but that all mathematical training is simply a question of teaching the subject thoroughly and in such a way that the student shall be able to use it freely. From the standpoint of mathematicians separate departments of mathematics for different classes of students are regarded as narrowing and hurtful to all concerned, and regret is expressed that specialized professional courses are occupying an increasingly large place in the curriculum to the exclusion of a more thorough grounding in broad fundamentals

METHODS OF INSTRUCTION.

The method of instruction ordinarily pursued is based on the use of a textbook. Instruction then proceeds by assigning definite portions of the text, with daily recitations on the portion assigned, usually in the form of solving the accompanying problems in the text. The chief differences in applying this method consist in the manner of securing the required amount of problem work, some instructors relying chiefly on blackboard work during the class period, while others require independent outside work. The textbook method has the advantage of being simple and easy to follow, and from the point of view of the instructor doubtless produces more uniform results than could be obtained otherwise. It is open to criticism, however, as being conventional and stereotyped, and from the student's subsequent record in technology it develops that he is usually tied to the range of problems, and even the notation, given in the text studied, and is deficient in the freedom of use which comes from an actual mastery of fundamental principles.

Models are seldom used in the classroom and a mathematical laboratory is practically unknown.

AIM AND RESULTS OF INSTRUCTION.

The returns indicate that in deference to the opinions of departments of technology an effort is made to differentiate between mathematical instruction given to engineers and to others. For engineers the aim is to prepare for their professional courses by insisting on accuracy in computation and emphasizing such principles as are of immediate and practical application. At the same time it is the general opinion that there is a broad cultural value in mathematical training which engineers almost wholly miss by reason of the limitations to which their work in mathematics is subject. The cultural value of mathematics is defined in this connection as consisting in the concentration of attention on definite concepts and their inferences.

There are exceptions to this view, however, the replies having the greatest weight of experience back of them asserting that when mathematics is properly taught, although limited to technical subjects, engineers profit largely by this training aside from the specific value of the subject as a professional tool. This opinion, expressed by several, is summarized in one reply as follows:

> Our successful students are gaining clearness and accuracy of thought, power of attack, and fertility of invention that place them far above men not thus trained. Almost invariably those students who have risen most rapidly to high positions of power and usefulness have been among the most brilliant mathematicians so far as they have chosen to cultivate mathematical subjects.

PREPARATION OF TEACHERS.

The consensus of opinion on this point is that it is desirable that teachers of mathematics in technological schools shall have had some technical training or experience, or at least be interested in the technical and practical applications of mathematics. It is not considered practicable, or even desirable, to secure teachers who have had actual engineering experience. One reason advanced is that the pecuniary rewards in engineering practice are so much greater than in teaching that it is impossible to secure desirable experienced engineers as teachers. It is also thought that practical experience extending over a number of years would actually disqualify one for teaching by putting him out of touch with college life and out of sympathy with the best educational methods and ideals. It is admitted, however, that the teacher of mathematics to engineers should have an adequate knowledge of the applications of the subject, as otherwise he would be out of sympathy with the aims of the school.

The cause of the difficulty experienced in securing satisfactory teachers in this field probably lies in the natural scarcity of men who prefer teaching to professional work, the preference naturally connecting itself with aptitude.

The most efficient training for a teacher of mathematics in a technological school is believed to consist of a technical collegiate education, or academic course with the election of numerous technical courses involving the applications of mathematics, followed by graduate work in pure mathematics. Previous teaching experience is thought to be desirable but not essential.

It is emphatically asserted that mathematics should be taught by skilled mathematicians. The argument is that greater technical skill is required in mathematics than in any department of technology, and if the latter requires the services of specialists, mathematics can also be best imparted by experts in this particular line.

SUPPLEMENT ON SPECIAL METHODS OF INSTRUCTION.

I. THE TWO-HOUR SYSTEM AT HARVARD UNIVERSITY.

E. V. Huntington.

In certain courses in mathematics for undergraduates at Harvard University a special plan of instruction is followed which meets several important requirements in a large university.

Each large course (50 to 100 students) is in charge of a permanent member of the staff, assisted by two or more instructors recruited each year from the ranks of the graduate students. The course meets three times a week for two consecutive hours. In the first part of the period the professor in charge lectures to the entire class, explaining the method for solving the problems of the day. In the second part of the period the class moves into a large drafting room, where the students work out their problems under the immediate supervision of the instructors. That is, the three instructors, including the professor in charge, pass about the room from man to man, giving each man a brief individual conference on his work while the work is being done. These conferences are partly in the nature of a quiz, to make sure that the student is able to explain intelligently what he is doing; partly to give assistance over special individual difficulties; and partly to record the problem work. For this last purpose special individual record cards are kept. With these cards in his hand, the instructor has the complete record of each student before him when he approaches that student for his conference. No problems are "handed in" to be corrected out of hours; all the problem work done by the student remains in his notebook and is inspected and recorded by the instructor in his presence.

The work in the drafting room is quite informal, and the students are at liberty to discuss their problems with their neighbors. An essential feature of the plan, therefore, is a series of frequent short tests, usually every two weeks, in which each student works by himself. The marks for the course are chiefly based on these tests and on the usual mid-year and final examinations.

The chief advantages of the plan are that any difficulties the student may experience are cleared up at once; the instructor is enabled to keep in close and constant touch with the work of the class; and the university is able to avail itself of the services of experienced teachers who are taking graduate work on leave of absence from their own institutions, and are thus eligible for such temporary appointments.

II. REDIRECTION OF EDUCATIONAL AIMS.

S. E. Slocum.

From the facts brought out by the questionnaire it would seem that mathematical instruction in technological departments of colleges and universities has not yet diverged very widely from the lines laid down as orthodox half a century ago or more, and originally intended as preparatory for one of the so-called learned professions. The rapid industrial development of recent years, however, has created a demand for a highly efficient and specialized form of technical training, in response to which a new type of school has arisen, known variously as trade, technical, manual, vocational, commercial, industrial, continuation or cooperative. This has been accompanied by a radical revision of academic standards, the popular ideal, at least, being that of a training which fits directly for industrial and social efficiency. This is evidenced not only in individual efforts at modification of the curriculum in many of our own institutions, but also in recent government legislation in Great Britain, France, and Germany.

The appointment of this commission is, in itself, a recognition of the fact that a new educational situation has arisen which demands special consideration, and the value of its reports will consist largely in informing teachers generally of the trend of our American educational policy and in harmonizing cultural ideals with the practical demand for industrial and social efficiency. To further supplement the returns to the questionnaire it therefore seems desirable to point out the most salient features of the new problem in technological education as apparent in the scheme of industrial cooperation, with special reference to its practical effect on mathematical instruction.

THE COOPERATIVE SYSTEM AT THE UNIVERSITY OF CINCINNATI.

The effect of any popular propaganda, such as the demand for vocational efficiency referred to above, is of course first apparent in secondary education, but, although of comparatively recent origin, the movement already shows an outcropping at the university level in several places, notably in the cooperative system established at the Lewis Institute, Chicago, and at the University of Cincinnati. The basis of this innovation is usually supposed to consist in the simple fact of mechanical alternation between school and shop, whereby a manual as well as an intellectual development is secured, and a trade acquired simultaneously with a profession. A more adequate and characteristic summary of the ideas underlying the cooperative system, however, is that given in the following extract from a recent article on the subject:

> A definite line of communication has been established from the school to the forge and shop and railroad; from the laboratory and lecture hall, where the laws underlying and governing the work are formulated and made clear, to where men actually face

the problems of progress. The embryo engineer travels back and forth between the two weekly, and always faces the task before he is brought to the science. The laws of nature, then, have for him an increased significance. Literature and the humanities have life. Fact and theory are in constant juxtaposition. That strength is gained which we call character, and a safe mental development based on physical effort is obtained in conformity to nature's laws. "Going to college" every other week from the workshop has a different meaning from "going to college" once from the high school. It becomes an opportunity that is denied other men in overalls and jumpers, and a corresponding feeling of indebtedness to the one not so favored develops. It means future cooperation rather than future misunderstanding and mutual distrust. We have applied the term "cooperative system" to this method of training as much in hope of its future results as in description of the method.

EFFECT OF THE COOPERATIVE SYSTEM ON MATHEMATICAL INSTRUCTION.

It is obvious that such an intimate correlation of the concrete with the abstract is bound to affect both the form and content of instruction, especially in the case of a basic science like mathematics. As an instance of this, the result of five years' operation of the cooperative system at the University of Cincinnati shows an increasing tendency to break down the barriers between departmental instruction in mathematics and allied subjects, particularly mechanics. Increasing emphasis is thereby laid upon mathematics as a general method, or, from another point of view, as the unifying principle underlying all branches of technology. The practical development of this idea in mathematical instruction in connection with the cooperative system has been gradual, accompanying the growth of the general scheme of correlation of departments and shops. Sufficient progress has already been made, however, to show that eventually the principles of elementary mechanics will be used for unifying mathematical instruction and correlating it with the manual experience of the student, as well as with other branches of technology. Effectual provision will thus be made for laboratory instruction in mathematics, long thought to be desirable, but difficult of satisfactory attainment.

In mechanics, which is taught by a separate department of applied mathematics, the cooperative viewpoint was adopted at the outset, although improvements in method are still being made. In detail, the method consisted primarily in urging each cooperative student to bring to the classroom for solution each week some practical problem in mechanics or the strength of materials which had come under his observation in the shop during the previous week. This simple plan for securing the active cooperation of the student in his own instruction put him in a receptive mental attitude, and at the same time greatly increased the practical and informational value of the course. By this method, individualizing of instruction, which hitherto has been an ideal rather than a reality, appears spontaneously, without effort on the part of the teacher, and the elimination of memory work is accompanied by a concentration of attention on methods and processes rather than their outcome. The most important features of this plan are that the student and his activities become the center of interest and his intellectual and manual activities so articulated as to act as mutual stimuli.

There can be no doubt that the current of modern educational thought is setting strongly in the direction of practical cooperation such as here indicated, as shown, for instance, by the results of the investigation carried out by the Manuel Général in France, and the impression which such men as Dr. Kerschensteiner, of Munich, are making upon the educational policy of Germany. The changes advocated, however, are not as radical as may appear at first sight, and no fear need be entertained as to the loss of any of the so-called cultural value of mathematical instruction if by culture is understood a well-rounded development of the individual rather than intellectual conformity to a rigid Procrustean standard.

CONCLUSION.

The facts here presented indicate that orthodox instruction in mathematics does not entirely harmonize with present ideals of material development. In justice to present methods it should be said, however, that this by no means implies that instruction in mathematics is less efficient than in other subjects. A comparison of results with those obtained in other departments, such as English or physics, will show no greater returns for a given expenditure of time and energy. The greater prevalence of criticism of mathematical instruction arises chiefly from the fact that any deficiency in this subject is more apparent than in others, as mathematics precedes technological subjects and serves as their basis. Moreover, the inability to use mathematical principles once taught may be more apparent than real, since it will often be found that when these principles are needed for application a brief explanation by the instructor in technology will suffice to recall them with all their original force.

There is perhaps more ground for the criticism that mathematical instruction at present is too formal and better suited to the needs of a specialist in mathematics than to a student of technology. The idea of formal discipline as an educational aim is, in fact, a thing of the past. A sufficient evidence of its futility in the present case is the difficulty which the average student experiences in making even simple technical applications of mathematical principles in which he has been carefully drilled. On the other hand, the fear that technical applications will lessen the cultural value of mathematics is groundless, as a principle or problem loses none of its theoretical importance because it also has a human or economic value. For instance, in such a prosaic field as shop management the well-known Taylor system has resulted in the development of a slide rule for handling 12 variables simultaneously, and the fact that its use increases the output of a machine 50 per cent by no means lessens its importance as an achievement of mathematical genius.

The whole question seems to resolve itself into the problem of making mathematics more expressive of modern conditions. Herbart asserts that interest is of fundamental pedagogical importance, and that interest is based on apperception, by which he means that no instruction can be assimilated until its relation to concepts already familiar is fully apparent. This indicates that even from a purely pedagogical standpoint mathematical instruction should proceed slowly and each new idea be followed out into as many of its ramifications as possible in order to provide the connective tissue needed to make education organic. It is suggested, therefore, that effort be directed toward making instruction in mathematics cumulative; that is to say, instead of placing the chief emphasis on the logical development of the subject and relying for practice on sets of unrelated problems illustrating merely a single phase or principle, its importance as the unifying principle underlying all exact methods of expression should be made apparent by a series of original applications of increasing complexity, each of which shall embody as much as possible of all preceding instruction in the subject. A graded series of practical problems in mechanics and engineering design arising out of the student's growing experience is typical of what is meant. Less ground might apparently be covered in this way, but there can be no doubt that the ordinary student would make more substantial and lasting progress.

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